Kinetic Modulational Instability of Upper-hybrid Turbulence

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It is shown that a plasma containing randomly distributed non-interacting upper-hybrid waves can become unstable against ion quasi-modes. The growth rate of the instability is presented.

In previous papers [1, 2], it has been shown that the upper-hybrid turbulence consisting of an ensemble of random-phased upper-hybrid waves can become modulationally unstable with respect to low-frequency ion-cyclotron, lower-hybrid [1], and adiabatic perturbations [2]. The latter is valid for the quasi-static régime.

In this letter, we consider the problem of modulation of upper-hybrid turbulence by low-frequency perturbations (Ω, q). A general dispersion relation which is valid for the quasi-static $(|\Omega/q\, v_{\mathrm{Ti}}| \ll 1)$, the inertial $(|\Omega/q\, v_{\mathrm{Ti}}| > 1)$, and the transitional $(|\Omega/q\, v_{\mathrm{Ti}}| \sim 1)$ regimes is obtained. Specificially, we shall be concerned with the last régime, namely the problem of coupling of upper-hybrid turbulence with ion quasi-modes. For the latter type of perturbations, the two fluid description fails and one should retain the Vlasov description.

In what follows, the high-frequency upper-hybrid waves shall be allowed to have a small component E_z along the external magnetic field $B_0 \hat{z}$ so that they may couple with the ion quasi-modes. The dynamics of the upper-hybrid turbulence is governed by a wave kinetic equation [1, 3]. The change of the upper-hybrid turbulence distribution $\tilde{N}_k$ in the presence of ion quasi-mode perturbations $\tilde{n}_e$ is given by [1]

$$\tilde{N}_k = -\frac{\omega_k}{2}\frac{\tilde{n}_e}{n_0}\, q\cdot \frac{\partial N_k{}^0/\partial k}{\Omega - q\cdot v_g}, \qquad (1)$$

where $N_k{}^0 = \langle |E_k{}^0|^2\rangle / 4\pi\omega_k$ is the unperturbed distribution, and

$$\omega_k^2 = \omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2 + 3\, k^2 v_{\mathrm{Te}}^2\, \omega_{\mathrm{pe}}^2/(\omega_{\mathrm{pe}}^2 - 3\,\omega_{\mathrm{ce}}^2)$$

is the characteristic frequency of the turbulence. Here v_{Te}, ω_{pe}, and ω_{ce} are respectively the thermal velocity, plasma and gyrofrequencies of the electrons. The group velocity of the high-frequency waves is given by

$$v_g = \hat{x}\,[3\, k\, \lambda_e\, v_{\mathrm{Te}}\, \omega_{\mathrm{pe}}/\omega_k\, (1 - 3\,\omega_{\mathrm{ce}}^2/\omega_{\mathrm{pe}}^2)]\,, \qquad (2)$$

where $\lambda_e = v_{\mathrm{Te}}/\omega_{\mathrm{pe}}$ is the electron Debye length. We note that for $3\,\omega_{\mathrm{ce}}^2 \ll \omega_{\mathrm{pe}}^2$, the upper-hybrid modes have positive group dispersion, whereas for $\omega_{\mathrm{pe}}^2 \ll 3\,\omega_{\mathrm{ce}}^2$, the modes have negative group dispersion. In the following, we consider only the upper-hybrid waves with positive group dispersion, namely $3\,\omega_{\mathrm{ce}}^2 \ll \omega_{\mathrm{pe}}^2$.

The charge density perturbation ϱ_e in the presence of the modified distribution (1) is obtained from the electron Vlasov equation. We find

$$4\pi\varrho_e = -q^2 \chi_e\,(\Phi + \Phi_{\mathrm{pe}}) \qquad (3)$$

where Φ is the ambipolar potential, and the ponderomotive potential Φ_{pe} is given by

$$\Phi_{\mathrm{pe}} = -\sum_k \pi\, e\, \omega_k\, \tilde{N}_k / m_e\,(\omega_k^2 - \omega_{\mathrm{ce}}^2)\,. \qquad (4)$$

In (3), $\chi_e = (q\lambda_e)^{-2}\, G'(\Omega/q\, v_{\mathrm{Te}})$ is the electron susceptibility, and G is the plasma dispersion function. The electrons are assumed to be highly magnetized.

From the linearized ion Vlasov equation, we obtain

$$4\pi\varrho_i = -q^2\chi_i\,\Phi\,, \qquad (5)$$

where $\chi_i = (q\lambda_i)^{-2}\, G'(\Omega/q\, v_{\mathrm{Ti}})$ is the ion susceptibility, and ions are assumed to be unmagnetized. The ponderomotive potential Φ_{pi} of the ions is smaller than that of the electrons by a factor m_e/m_i and is therefore neglected.

Combining (3) and (5), and using Poisson's equation $q^2\Phi = 4\pi(\varrho_i + \varrho_e)$, one gets

$$\Phi = -\chi_e\,\Phi_{\mathrm{pe}}/\varepsilon\,, \qquad (6)$$

where $\varepsilon = 1 + \chi_e + \chi_i$. Inserting (6) into (3), we get

$$\frac{\tilde{n}_e}{n_0} = \frac{q^2\chi_e\,(1+\chi_i)}{4\pi\, n_0\, e\,\varepsilon}\,\Phi_{\mathrm{pe}}\,. \qquad (7)$$

Combining (1) and (7), we obtain the dispersion relation

$$1 = \frac{L}{16\pi n_0}\,\frac{q^2\chi_e\,(1+\chi_i)}{m_e\,\varepsilon}\,\frac{(\omega_{\mathrm{pe}}^2 + \omega_{\mathrm{ce}}^2)}{\omega_{\mathrm{pe}}^2} \cdot \int \frac{q\cdot\partial N_k{}^0/\partial k}{(\Omega - q\cdot v_g)}\,\mathrm{d}k\,, \qquad (8)$$

where the summation over k has been replaced by an integration in the usual manner (i.e., $\sum_k \mathrm{d}k \to L/2\pi \int \mathrm{d}k$, where L is the size of the system). Equa-

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tion (8) is the most general dispersion relation describing the interaction of upper-hybrid turbulence with low-frequency perturbations.

For $|\Omega| \ll q v_{Te}$, we have $\chi_e = (q \lambda_e)^{-2}$. We now let

$$N_k^0 \approx (2\pi)^{1/2} (W/4\pi\omega_{pe}) (k_t L)^{-1} \cdot \exp[-(k-k_0)^2/2k_t^2]$$

in (8) and assume that the spectrum of upper-hybrid turbulence is sufficiently peaked around k_0 [i.e., $|\Omega - q_x u_0| > 3 q_x \lambda_e k_t v_{Te} \omega_{pe}/(\omega_{pe}^2 + \omega_{ce}^2)^{1/2}$, where $u_0 = v_g(k = k_0)$], we then obtain from (8)

$$\Omega - q_x u_0 = \pm i (3/8\pi)^{1/2} q_x \lambda_e \omega_{pe} \cdot \left(\frac{W}{n_0 T_e}\right)^{1/2} \left(\frac{1+\chi_i}{\varepsilon}\right)^{1/2} A, \quad (9)$$

where k_t is the spread, k_0 is the mean wave vector, W is the total energy of the turbulence spectrum, and $4A^2 = \omega_H \omega_{pe}/(\omega_{pe}^2 - 3\omega_{ce}^2)$ with $\omega_H^2 = \omega_{pe}^2 + \omega_{ce}^2$.

Letting $\Omega = q_x u_0 + i\gamma$, we obtain the growth rate

$$\gamma = (3/8\pi)^{1/2} q_x \lambda_e (W/n_0 T_e)^{1/2} \omega_{pe} A R_e \cdot \left[\left(\frac{1+\chi_i}{\varepsilon}\right)\right]^{1/2}_{\Omega = q_x u_0}, \quad (10)$$

where the frequency shift caused by the turbulence is neglected. When the argument of χ_i is near unity, that is $3 k_0 \lambda_e v_{Te} \approx v_{Ti}$, which can occur for $k_0 \lambda_e \ll 1$, we find for $T_e \approx T_i$,

$$\gamma \approx 0.9 (3/8\pi)^{1/2} q_x \lambda_e (W/n_0 T_e)^{1/2} \omega_{pe} A. \quad (11)$$

In conclusion, we have shown that a spectrum of upper-hybrid mode turbulence is unstable when the group velocity of the turbulence mode is approximately equal to the ion thermal speed.

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